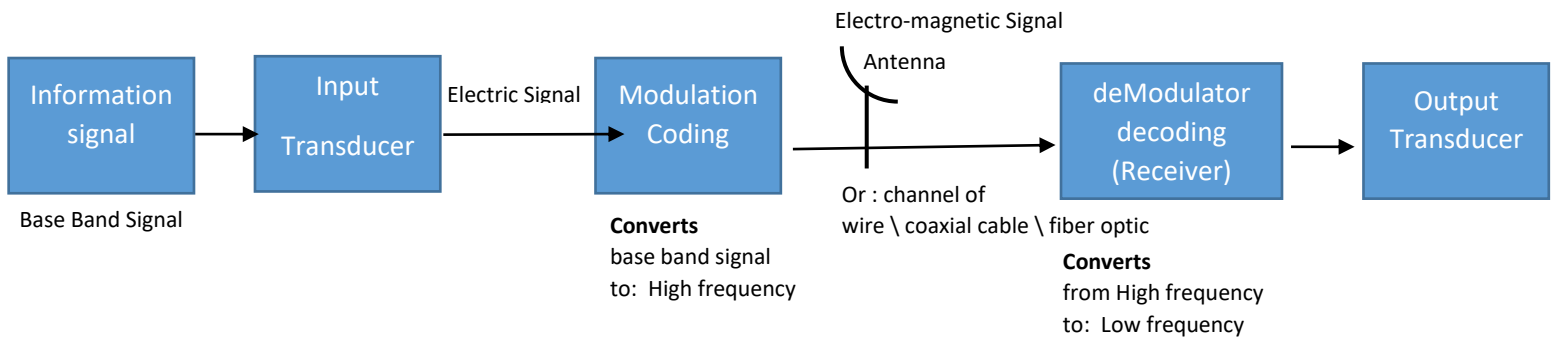


Communication

Elements of Communication System

Diagram



Modulation Types

- AM Modulation
- FM Modulation
- PM Modulation

Signal Analysis (Fourier Analysis)

Conditions for Fourier Analysis (Dirichlets Conditions)

- 1- Integrable $\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} g_p(t) dt < \infty$
- 2- $g_p(t)$ has finite number of maximums and minimums
- 3- $g_p(t)$ has a finite number of discontinuities

Sin and Cos are Orthogonal

$$\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \sin\left(\frac{2\pi mt}{T_0}\right) \cos\left(\frac{2\pi nt}{T_0}\right) dt = 0 \quad , for all \quad m, n$$

$$\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \cos\left(\frac{2\pi mt}{T_0}\right) \cos\left(\frac{2\pi nt}{T_0}\right) dt \begin{cases} = \frac{T_0}{2} & \text{if } m = n \\ = 0 & \text{if } m \neq n \end{cases}$$

Fourier (Periodic Signal)

Sines and Cosines	Complex or Exponential form
$f(t) = a_0 + 2 \sum_{n=1}^{\infty} [a_n \cos n\omega t + b_n \sin n\omega t]$ <p style="text-align: center;"><u>Where :</u></p> <p><i>if even</i> $\rightarrow b_n = 0$ <i>if odd</i> $\rightarrow a_n = 0$</p> $\omega = \frac{2\pi}{T} = 2\pi f$ $a_0 = \frac{1}{T} \int_T f(t) dt$ $a_n = \frac{1}{T} \int_T f(t) \cos n\omega t dt$ $b_n = \frac{1}{T} \int_T f(t) \sin n\omega t dt$	$f(t) = \sum_{-\infty}^{\infty} C_n e^{jn\omega t}$ <p style="text-align: center;"><u>Where</u></p> $C_n = \frac{1}{T} \int_T f(t) e^{-jn\omega t} dt$ $ C_n = \sqrt{a_n^2 + b_n^2}$

Power Average

$$P_{av} = \frac{1}{T} \int (f(t))^2 dt$$

PROOF (Parseval' s Power Therorem)

for load = 1Ω [Normalized Power] , f(t): voltage or current

$$|f(t)| = \sqrt{(\text{real})^2 + (\text{img})^2}$$

$$|f(t)|^2 = f(t) * f^*(t)$$

$$P = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} f(t) * f^*(t) dt$$

$$f(t) = \sum_{-\infty}^{\infty} C_n e^{jn\omega t}$$

$$P = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \sum_{-\infty}^{\infty} C_n e^{jn\omega t} * f^*(t) dt$$

$$P = \frac{1}{T_0} * \sum_{-\infty}^{\infty} C_n \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} e^{jn\omega t} * f^*(t) dt$$

$$P = \sum_{-\infty}^{\infty} C_n * \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} e^{jn\omega t} * f^*(t) dt$$

$$\therefore C_n = \frac{1}{T} \int_T f(t) e^{-jn\omega t}$$

$$\therefore C_n^* = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} e^{jn\omega t} * f^*(t) dt$$

$$\therefore P = \sum_{n=-\infty}^{\infty} C_n C_n^* \quad (\text{Parseval's Power Theorem})$$

$$P_{av} = \frac{1}{T} \int |f(t)|^2 dt$$

Parseval's Power Theorem

Normalized Value of the mean Power for a periodic signal is the sum of the squared amplitude of all harmonics in the signal

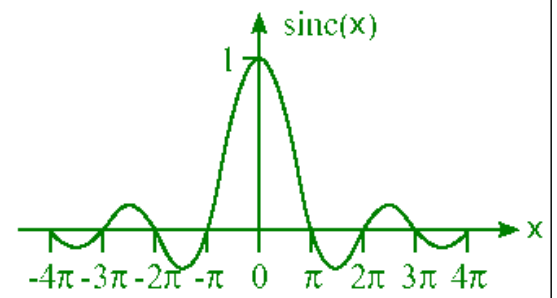
Energy Content of a finite signal

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt \quad [\text{Normalized Energy}]$$

$$E = \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega \quad [\text{Parseval's Theorem}]$$

Signal Spectrum & Sinc Function

If you graph C_n you get signal spectrum as a sinc fun. $\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$



Fourier (Non-Periodic Signal)

Fourier :

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$\text{or) } G(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} dt$$

Fourier Inverse :

$$g(t) = \int_{-\infty}^{\infty} G(f) e^{j2\pi ft} df$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

Famous Functions (time / Freq)

$g(t)$	$G(f) = \int_{-\infty}^{\infty} g(t)e^{-j2\pi ft} dt$	$G(\omega) = \int_{-\infty}^{\infty} g(t)e^{-j\omega t}$
$rect(t)$	Tsinc(fT)	$Tsinc\left(\frac{\omega T}{2\pi}\right)$
$Arect\left(\frac{t}{T}\right)$	ATsinc (fT)	$AT sinc\left(\frac{\omega}{2\pi}\right)$
$tri(t)$	sinc²(f)	$sinc^2\left(\frac{\omega}{2\pi}\right)$
$ATtri\left(\frac{t}{T}\right)$	AT²sinc²(fT)	
C (const)	Cδ(f)	$2\pi C\delta(\omega)$
$\delta(t)$	1	1
$\delta(t - a)$	e^{-j2πfa}	$e^{-j\omega a}$
$\cos(2\pi At)$	$\frac{1}{2} [\delta(f - A) + \delta(f + A)]$	$\pi [\delta(\omega - 2\pi A) + \delta(\omega + 2\pi A)]$
$\sin(2\pi At)$	$\frac{1}{2j} [\delta(f - A) - \delta(f + A)]$	$\frac{\pi}{j} [\delta(\omega - 2\pi A) - \delta(\omega + 2\pi A)]$
$sgn(t) = 2u(t) - 1$	$\lim_{\alpha \rightarrow 0} \frac{1}{\alpha + j2\pi f} - \frac{1}{\alpha - j2\pi f} = \frac{1}{j\pi f}$	$\frac{2}{j\omega}$
$u(t)$ (step)	$\frac{1}{j2\pi f} + \frac{\delta(f)}{2}$	$\frac{1}{j\omega} + \pi \delta(\omega)$
$e^{-\pi t^2}$ (Gaussian)	$e^{-\pi f^2}$	$e^{-\left(\frac{\omega^2}{4\pi}\right)}$
$e^{-\alpha t} u(t)$	$\frac{1}{\alpha + j2\pi f}$	
$e^{\alpha t} u(-t)$	$\frac{1}{\alpha - j2\pi f}$	
$e^{-\alpha t }$	$\frac{1}{\alpha + j2\pi f} + \frac{1}{\alpha - j2\pi f}$ $= \frac{2\alpha}{\alpha^2 + 4\pi^2 f^2}$	
$g_P(t)$	$\sum_{-\infty}^{\infty} C_n \delta\left(f - \frac{n}{T_o}\right)$	
INVERSE	Inverse $g(t) = \int_{-\infty}^{\infty} G(f)e^{j2\pi ft} df$	Inverse $g(t) = \int_{-\infty}^{\infty} G(\omega)e^{j\omega t} d\omega$

$$\cos(A + B) = \cos(A) \cos(B) - \sin(A) \sin(B)$$

$$\sin(A + B) = \sin(A) \cos(B) + \cos(A) \sin(B)$$

$$\cos(A) \cos(B) = \frac{1}{2} [\cos(A - B) + \cos(A + B)]$$

$$\sin(A) \sin(B) = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$$

$$\sin(A) \cos(B) = \frac{1}{2} [\sin(A + B) + \sin(A - B)]$$

$$\cos(A) \sin(B) = \frac{1}{2} [\sin(A + B) - \sin(A - B)]$$

$$\cos^2 \theta = \frac{1}{2} (1 + \cos 2\theta)$$

$$\cos^3 \theta = \frac{1}{4} (\cos 3\theta + 3\cos \theta)$$

Properties of Fourier Transform

1- Linearity (Super Position)	2- Time Scaling
$a_1x_1(t) + a_2x_2(t) \Rightarrow a_1X_1(f) + a_2X_2(f)$	$x(at) \Rightarrow \frac{1}{ a } x\left(\frac{f}{a}\right)$
3- Time Shifting نفس	4- Frequency Shifting عكس
$x(t - t_0) \Rightarrow X(f)e^{-j2\pi f t_0}$ $x(t + t_0) \Rightarrow X(f)e^{j2\pi f t_0}$	$x(t)e^{j2\pi f_0 t} \Rightarrow X(f - f_0)$
5- Time-Reversal	6- Duality
$x(-t) \Rightarrow X(-f)$	$G(t) \Rightarrow g(-f)$
7- Differentiation	8- Integration
$\frac{d}{dt} g(t) \Rightarrow j2\pi f G(f)$	$\int_{-\infty}^{\infty} g(t) dt \Rightarrow \frac{1}{j2\pi f} G(f) + \frac{G(0)}{2} \delta(f)$
9- Mult. In time domain (Modulation)	10- Convolution
$f\{x(t)y(t)\} \Rightarrow X(f) \otimes Y(f)$ $= \int_{-\infty}^{\infty} X(\lambda)Y(f - \lambda)d\lambda$	$x(f) \otimes y(f) \Rightarrow X(f)Y(f)$
11- Conjugate	
$g^*(t) \Rightarrow G^*(-f)$	
12- Area Under g(t)	13- Area under G(f)
$G(0) = \int_{-\infty}^{\infty} g(t)dt$	$G(0) = \int_{-\infty}^{\infty} G(f)df$

Famous Examples (Proofs)

RECT

Find Fourier for $g(t) = A \text{rect}\left(\frac{t}{T}\right)$

Solution

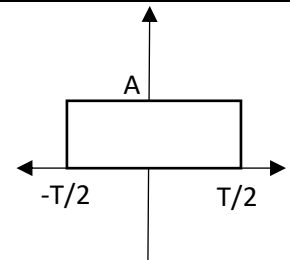
$$G(f) = \int_{-\frac{T}{2}}^{\frac{T}{2}} g(t) e^{-j\omega t} dt$$

$$G(f) = \int_{-\frac{T}{2}}^{\frac{T}{2}} A e^{-j\omega t} dt$$

$$\omega = 2\pi f$$

$$G(f) = \frac{A}{\pi f} \left[\frac{e^{j\pi f T} - e^{-j\pi f T}}{2j} \right]$$

$$= A \frac{\sin(\pi f T)}{\pi f} * \frac{T}{T} = AT \text{sinc}(fT)$$



TRI

$$\Delta(f) = \mathfrak{F}\{\Lambda(t)\} = \int_{-\infty}^{\infty} \Lambda(t) e^{-2\pi i f t} dt$$

$$= \int_{-1}^0 (1+t) e^{-2\pi i f t} dt + \int_0^1 (1-t) e^{-2\pi i f t} dt$$

$$= \left[\frac{1+2\pi i f}{4\pi^2 f^2} - \frac{e^{2\pi i f}}{4\pi^2 f^2} \right] - \left[\frac{2\pi i f - 1}{4\pi^2 f^2} + \frac{e^{-2\pi i f}}{4\pi^2 f^2} \right]$$

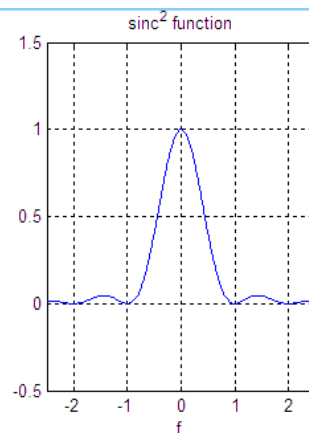
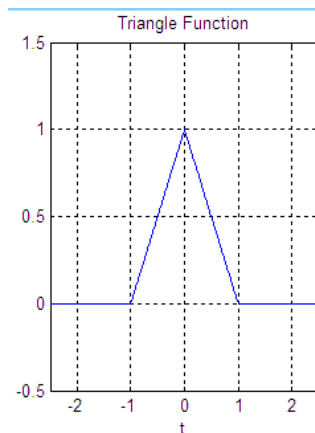
$$= -\frac{e^{-2\pi i f} (e^{2\pi i f} - 1)^2}{4\pi^2 f^2}$$

$$= -\frac{e^{-2\pi i f} (e^{\pi i f} [e^{\pi i f} - e^{-\pi i f}])^2}{4\pi^2 f^2}$$

$$= -\frac{e^{-2\pi i f} e^{2\pi i f} (2i)^2 \sin^2(\pi f)}{4\pi^2 f^2}$$

$$= \left(\frac{\sin(\pi f)}{\pi f} \right)^2 = \text{sinc}^2 f$$

$$\Lambda(t) = \begin{cases} 1-|t|, & |t| \leq 1 \\ 0, & \text{otherwise} \end{cases}$$



Handwritten derivation of the Fourier transform of a rectangular pulse and its autocorrelation:

$g_1(t) = A \text{rect}\left(\frac{t}{T}\right)$
 $\text{rect}\left(\frac{t}{T}\right) \Rightarrow AT \text{sinc}(fT)$
 $\Rightarrow g_1(f) \Rightarrow AT \text{sinc}(fT) \cdot e^{j\pi f T/2} - AT \text{sinc}(fT) \cdot e^{-j\pi f T/2}$
 $G_1(f) = 2j AT \text{sinc}(fT) \left[\frac{e^{j\pi f T} - e^{-j\pi f T}}{2j} \right]$
 $G_1(f) = 2j AT \text{sinc}(fT) \cdot \sin(\pi f T)$
 $\Rightarrow g_2(t) = \int_{-T}^T g_1(t) dt$
 $G_2(f) = \frac{1}{j 2\pi f} G_1(f) = AT^2 \text{sinc}(fT) \cdot \frac{\sin(\pi f T)}{\pi f T}$
 $= AT^2 \text{sinc}^2(fT)$

$$f(t) = \delta(t - a)$$

$$G(f) = \mathfrak{F}\{\delta(t - a)\} = \int_{-\infty}^{\infty} \delta(t - a) e^{-i2\pi ft} dt$$

$$= e^{-i2\pi fa}$$

Cos and Sin

$$G(f) = \mathfrak{F}\{\cos(2\pi At)\} = \int_{-\infty}^{\infty} \frac{e^{i2\pi At} + e^{-i2\pi At}}{2} e^{-i2\pi ft} dt$$

$$= \frac{1}{2} \left[\int_{-\infty}^{\infty} e^{i2\pi At} e^{-i2\pi ft} dt + \int_{-\infty}^{\infty} e^{-i2\pi At} e^{-i2\pi ft} dt \right]$$

$$= \frac{1}{2} [\delta(f - A) + \delta(f + A)]$$

$$\sin(2\pi At) = \frac{e^{i2\pi At} - e^{-i2\pi At}}{2i}$$

$$\mathfrak{F}\{\sin(2\pi At)\} = \frac{1}{2i} [\delta(f - A) - \delta(f + A)]$$

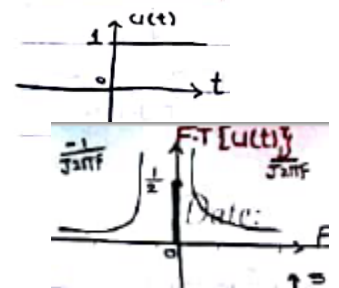
Unit Step

$$u(t) = \int_{-\infty}^t \delta(t) dt$$

$$\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & t \neq 0 \end{cases}$$

$$\text{Note : } \int \delta(t) = 1$$

$$\mathcal{F}\{u(t)\} \rightarrow \frac{1}{j2\pi f} + \frac{1}{2} \delta(f)$$



Sgnum

$$sgn = 2u(t) - 1$$

$$\mathcal{F}\{sgn\} = 2 \left[\frac{1}{j2\pi f} + \frac{1}{2} \delta(f) \right] - \delta(f) = \frac{1}{j\pi f}$$

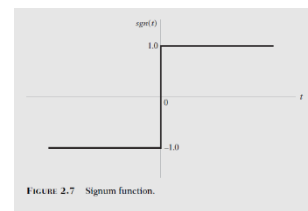


FIGURE 2.7 Signum function.

Finite Power Signal

$$\int_{-\infty}^{\infty} g(t) dt < \infty$$

$$\lim_{T \rightarrow 0} \int_{-\frac{T}{2}}^{\frac{T}{2}} g(t)^2 dt < \infty$$

Delta (impulse) function

$$1- \int_{-\infty}^{\infty} g(t) \cdot \delta(t) dt = g(0)$$

$$2- \int_{-\infty}^{\infty} g(t) \cdot \delta(t - t_0) dt = g(t_0)$$

$$3- \mathcal{F}\{\delta(t)\} = \int_{-\infty}^{\infty} \delta(t) e^{-j2\pi f t} dt = 1$$

$$4- \delta(t - t_0) \rightarrow e^{-j2\pi f t_0}$$

Gaussian

$$g(t) = \frac{1}{\tau} e^{-\frac{\pi t^2}{\tau}}$$

τ decrease then

t – domain : Amplitude \uparrow width \downarrow

f – domain : Amplitude \downarrow width \uparrow

any signal multiplied will shift it

τ decrease we get sharp pulse in t – domain

Periodic Signal

$$g_P(t) = \sum_{-\infty}^{\infty} C_n e^{\frac{j2\pi n t}{T_o}}$$

$$C_n = \frac{1}{T} \int_T g_P(t) e^{\frac{j2\pi n t}{T_o}}$$

$$\mathcal{F}\{g_P(t)\} = \sum_{-\infty}^{\infty} C_n \delta(f - \frac{n}{T_o})$$

Linearity

$$\mathcal{F}\{c_1g(t) + c_2h(t)\} = c_1G(f) + c_2H(f)$$

$$\begin{aligned}\mathcal{F}\{c_1g(t) + c_2h(t)\} &= \int_{-\infty}^{\infty} c_1g(t)e^{-i2\pi ft} dt + \int_{-\infty}^{\infty} c_2h(t)e^{-i2\pi ft} dt \\ &= c_1 \int_{-\infty}^{\infty} g(t)e^{-i2\pi ft} dt + c_2 \int_{-\infty}^{\infty} h(t)e^{-i2\pi ft} dt \\ &= c_1G(f) + c_2H(f)\end{aligned}$$

Time Scaling

$$\mathcal{F}\{g(ct)\} = \frac{G(\frac{f}{c})}{|c|}$$

if $c > 0$:

$$F\{g(ct)\} = \int_{-\infty}^{\infty} \frac{g(u)}{c} e^{-i2\pi f \frac{u}{c}} du = \frac{G(\frac{f}{c})}{c} = \frac{G(\frac{f}{c})}{|c|}$$

$$\begin{aligned}F\{g(ct)\} &= \int_{-\infty}^{\infty} g(ct)e^{-i2\pi ft} dt \\ \text{substitute: } u &= ct, \quad du = c dt \\ F\{g(ct)\} &= \int_{-\infty}^{\infty} \frac{g(u)}{c} e^{-i2\pi f \frac{u}{c}} du\end{aligned}$$

if $c < 0$:

$$\begin{aligned}F\{g(ct)\} &= \int_{\infty}^{-\infty} \frac{g(u)}{c} e^{-i2\pi f \frac{u}{c}} du = - \int_{-\infty}^{\infty} \frac{g(u)}{c} e^{-i2\pi f \frac{u}{c}} du \\ &= \frac{G(\frac{f}{c})}{-c} = \frac{G(\frac{f}{c})}{|c|}\end{aligned}$$

Time Shifting

$$\begin{aligned}\mathcal{F}\{g(t-a)\} &= \int_{-\infty}^{\infty} g(t-a)e^{-i2\pi ft} dt \\ &= \int_{-\infty}^{\infty} g(u)e^{-i2\pi f(u+a)} du \\ &= e^{-i2\pi fa} \int_{-\infty}^{\infty} g(u)e^{-i2\pi fu} du \\ &= e^{-i2\pi fa} G(f)\end{aligned}$$

Differentiating

$$F.T \quad G(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi f t} dt$$

$$I.F.T \quad g(t) = \int_{-\infty}^{\infty} G(f) e^{j2\pi f t} df$$

$$\therefore \frac{d}{dt} g(t) \Rightarrow j2\pi f G(f)$$

$$\text{and, } \frac{d^n}{dt^n} g(t) \Rightarrow (j2\pi f)^n G(f)$$

integration

Integration in the domain

$$\int_{-\infty}^t g(t) dt \Rightarrow \frac{1}{j2\pi f} G(f) \quad G(0) = 0$$

$$g(t) = \frac{d}{dt} \left[\int_{-\infty}^t g(t) dt \right]$$

$$G(f) = j2\pi f F \left\{ \int_{-\infty}^t g(t) dt \right\}$$

$$\therefore \int_{-\infty}^t g(t) dt \Rightarrow \frac{1}{j2\pi f} G(f)$$

$$\int_{-\infty}^t g(t) dt \Rightarrow \frac{1}{j2\pi f} G(f) + \frac{G(0)}{x} S(f) ??$$

Area under g(t)

$$\int_{-\infty}^{\infty} g(t) dt = G(0)$$

$$G(f) = \left[\int_{-\infty}^{\infty} g(t) e^{-j2\pi f t} dt \right]_{f=0}$$

$$G(0) = \int_{-\infty}^{\infty} g(t) dt$$

Area under G(f)

$$\int_{-\infty}^{\infty} G(f) df = g(0)$$

1) Duality

$$g(t) = \int_{-\infty}^{\infty} G(f) \cdot e^{+j2\pi ft} df$$

replace t by $-t$ in both sides

$$g(-t) = \int_{-\infty}^{\infty} G(f) \cdot e^{-j2\pi ft} df$$

replace t by f and f by t

$$\therefore g(-f) = \int_{-\infty}^{\infty} G(t) \cdot e^{-j2\pi ft} dt$$

$$\therefore G(t) \Leftrightarrow g(-f)$$

2) Time Scaling

$$F[g(at)] = \int_{-\infty}^{\infty} g(\underline{at}) \cdot e^{-j2\pi ft} dt \quad \xrightarrow{\boxed{1}}$$

$\frac{d\underline{at}}{dt} = a$

$$\text{let } \tau = at \rightarrow d\tau = a dt$$

$$t = \tau/a \quad dt = \frac{d\tau}{a}$$

عوض في $\boxed{1}$

$$\begin{aligned} F[g(at)] &= \frac{1}{a} \int_{-\infty}^{\infty} g(\tau) \cdot e^{-j2\pi f \frac{\tau}{a}} d\tau \\ &= \frac{1}{a} \cdot G\left(\frac{f}{a}\right) \end{aligned}$$

Conjugate

$$g^*(t) \Leftrightarrow G^*(-f)$$

$$g(t) = \int_{-\infty}^{\infty} G(f) \exp(j2\pi ft) df$$

$$g^*(t) = \int_{-\infty}^{\infty} G^*(f) \exp(-j2\pi ft) df$$

$-f$ gives

$$\begin{aligned} g^*(t) &= - \int_{\infty}^{-\infty} G^*(-f) \exp(j2\pi ft) df \\ &= \int_{-\infty}^{\infty} G^*(-f) \exp(j2\pi ft) df \end{aligned}$$

$$\textbf{Prove that } f\{x(t)y(t)\} = \int_{-\infty}^{\infty} X(\lambda)Y(F - \lambda)d\lambda$$

Solution

$$\text{let } g(t) = x(t)y(t)$$

$$\therefore G(f) = \int_{-\infty}^{\infty} x(t)y(t)e^{-i2\pi ft} dt$$

$$\int_{-\infty}^{\infty} y(t) \left[\int_{-\infty}^{\infty} X(\lambda) e^{i2\pi\lambda t} d\lambda \right] * e^{i2\pi ft} dt$$

$$= \int_{-\infty}^{\infty} y(t) \left[\int_{-\infty}^{\infty} X(\lambda) e^{-i2\pi(f-\lambda)t} d\lambda \right] dt$$

$$= \int_{-\infty}^{\infty} X(\lambda) \left[\int_{-\infty}^{\infty} y(t) e^{-i2\pi(f-\lambda)t} dt \right] d\lambda$$

$$= \int_{-\infty}^{\infty} X(\lambda)Y(F - \lambda) d\lambda$$

TABLE A6.1 *Fourier-Transform Theorems*

<i>Property</i>	<i>Mathematical Description</i>
1. Linearity	$ag_1(t) + bg_2(t) \Leftrightarrow aG_1(f) + bG_2(f)$ where a and b are constants
2. Dilation (time scaling)	$g(at) \Leftrightarrow \frac{1}{ a }G\left(\frac{f}{a}\right)$ where a is a constant
3. Duality	If $g(t) \Leftrightarrow G(f)$, then $G(t) \Leftrightarrow g(-f)$
4. Time shifting	$g(t - t_0) \Leftrightarrow G(f) \exp(-j2\pi ft_0)$
5. Frequency shifting	$\exp(j2\pi f_c t)g(t) \Leftrightarrow G(f - f_c)$
6. Area under $g(t)$	$\int_{-\infty}^{\infty} g(t) dt = G(0)$
7. Area under $G(f)$	$g(0) = \int_{-\infty}^{\infty} G(f) df$
8. Differentiation in the time domain	$\frac{d}{dt}g(t) \Leftrightarrow j2\pi fG(f)$
9. Integration in the time domain	$\int_{-\infty}^t g(\tau) d\tau \Leftrightarrow \frac{1}{j2\pi f}G(f) + \frac{G(0)}{2}\delta(f)$
10. Conjugate functions	If $g(t) \Leftrightarrow G(f)$, then $g^*(t) \Leftrightarrow G^*(-f)$,
11. Multiplication in the time domain	$g_1(t)g_2(t) \Leftrightarrow \int_{-\infty}^{\infty} G_1(\lambda)G_2(f - \lambda) d\lambda$
12. Convolution in the time domain	$\int_{-\infty}^{\infty} g_1(\tau)g_2(t - \tau) d\tau \Leftrightarrow G_1(f)G_2(f)$
13. Correlation theorem	$\int_{-\infty}^{\infty} g_1(t)g_2^*(t - \tau) dt \Leftrightarrow G_1(f)G_2^*(f)$
14. Rayleigh's energy theorem	$\int_{-\infty}^{\infty} g(t) ^2 dt = \int_{-\infty}^{\infty} G(f) ^2 df$

TABLE A6.2 *Fourier-Transform Pairs*

<i>Time Function</i>	<i>Fourier Transform</i>
$\text{rect}\left(\frac{t}{T}\right)$	$T \text{sinc}(fT)$
$\text{sinc}(2Wt)$	$\frac{1}{2W} \text{rect}\left(\frac{f}{2W}\right)$
$\exp(-at)u(t), \quad a > 0$	$\frac{1}{a + j2\pi f}$
$\exp(-a t), \quad a > 0$	$\frac{2a}{a^2 + (2\pi f)^2}$
$\exp(-\pi t^2)$	$\exp(-\pi f^2)$
$\begin{cases} 1 - \frac{ t }{T}, & t < T \\ 0, & t \geq T \end{cases}$	$T \text{sinc}^2(fT)$
$\delta(t)$	1
1	$\delta(f)$
$\delta(t - t_0)$	$\exp(-j2\pi f t_0)$
$\exp(j2\pi f_c t)$	$\delta(f - f_c)$
$\cos(2\pi f_c t)$	$\frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)]$
$\sin(2\pi f_c t)$	$\frac{1}{2j}[\delta(f - f_c) - \delta(f + f_c)]$
$\text{sgn}(t)$	$\frac{1}{j\pi f}$
$\frac{1}{\pi t}$	$-j \text{sgn}(f)$
$u(t)$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$
$\sum_{i=-\infty}^{\infty} \delta(t - iT_0)$	$\frac{1}{T_0} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_0}\right)$

Notes: $u(t)$ = unit step function

$\delta(t)$ = Dirac delta function

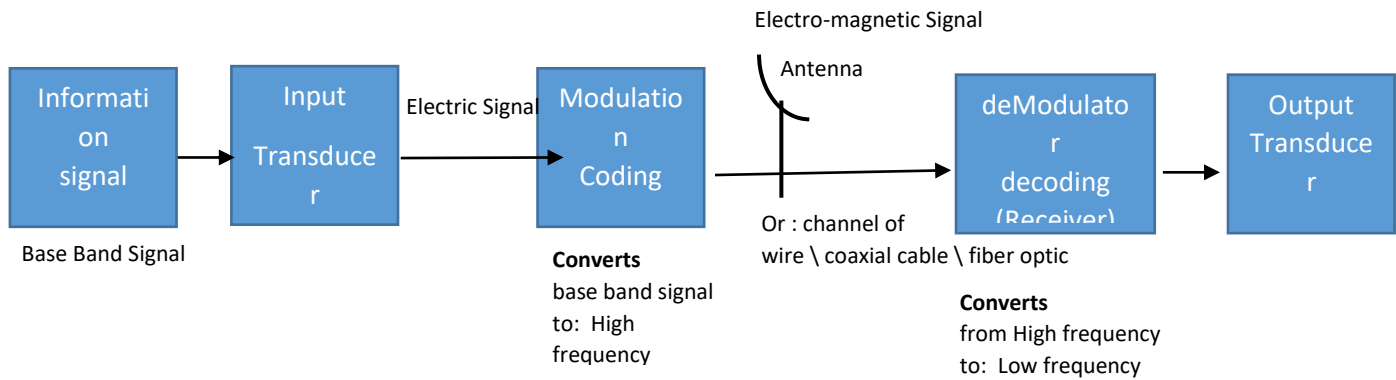
$\text{rect}(t)$ = rectangular function

$\text{sgn}(t)$ = signum function

$\text{sinc}(t)$ = sinc function

Amplitude Modulation

Communication System Diagram:



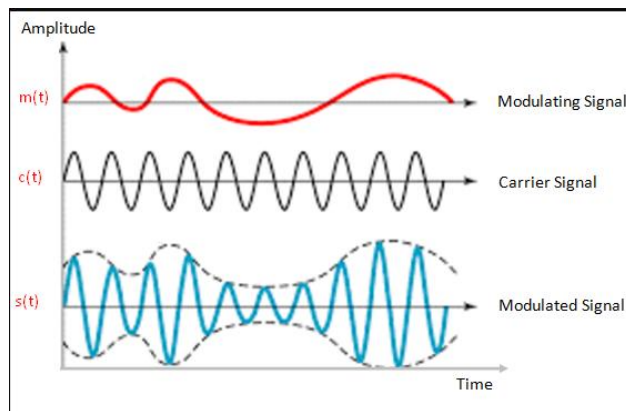
Base Band frequency (**300Hz : 3-4kHz**) “ will need a very long antenna” so we use a carrier
 Carrier Frequency (Very high) to use small antenna as $\lambda = \frac{c}{f}$

Modulation		
AM Wave 540-1600 kHz	FM Wave better smooth frequency 88-108 MHz	PM Wave

$$m(t) = A_m \cos(2\pi \overset{\text{small}}{\widetilde{f_m}} t) \quad ; \quad c(t) = A_c \cos(2\pi \overset{\text{large}}{\widetilde{f_c}} t)$$

after modulation : $s(t) = A_c [1 + K_a m(t)] \cos(2\pi f_c t)$

$K_a \text{ const} = \frac{1}{A_c}$ & $K_a m(t) = \frac{A_m}{A_c}$ **must be < 1** or it will overlap



To get Spectrum

$$s(t) = A_c \cos(2\pi f_c t) + A_c K_a m(t) \cos(2\pi f_c t)$$

$$S(f) = \frac{A_c}{2} [\delta(f - f_c) + \delta(f + f_c)] + \frac{A_c k_a}{2} [M(f - f_c) + M(f + f_c)]$$

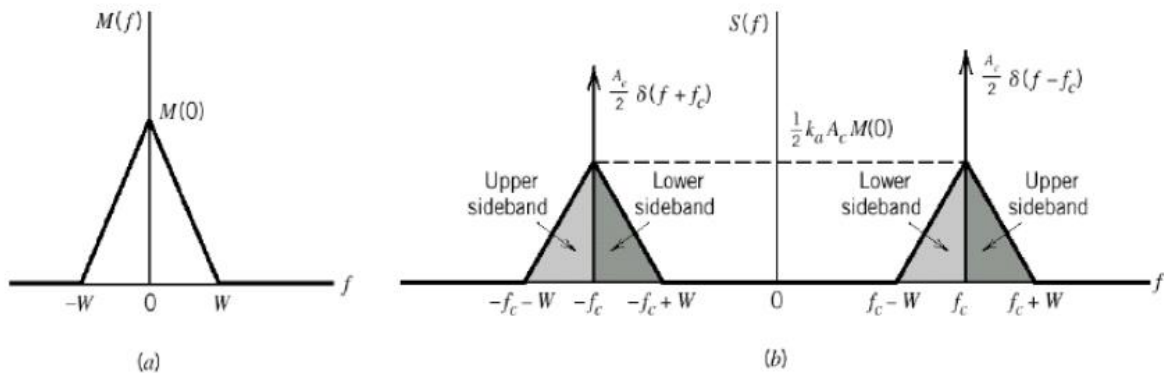


Figure 2.5: Spectrum of message and AM waves

μ Modulation Index

$$m(t) \xleftrightarrow{F.T.} M(f)$$

$$k_a m(t) \times 100 \rightarrow \text{Percentage modulation}$$

$$k_a |m(t)| = K_a A_m = \mu \quad (\text{modulation index}) = \frac{A_{Max} - A_{Min}}{A_{Max} + A_{Min}}$$

So $S(t)$ becomes

$$s(t) = A_c [1 + \mu \cos(2\pi f_m t)] \cos(2\pi f_c t)$$

$$s(t) = A_c \cos(2\pi f_c t) + \mu A_c \cos(2\pi f_m t) \cdot \cos(2\pi f_c t)$$

$$s(t) = A_c \cos(2\pi f_c t) + \mu A_c \left(\frac{1}{2} \overbrace{[\cos(f_c + f_m)]}^{\text{Upper Side Band}} + \frac{1}{2} \overbrace{[\cos(f_c - f_m)]}^{\text{Lower Side Band}} \right)$$

Power of $S(t)$

$$\text{Power of sinusoidal} = \frac{(\text{Amplitude})^2}{2}$$

$$\text{Power of } s(t) = \frac{A_c^2}{2} + \frac{1}{4} \mu^2 A_c^2 \left(\frac{1}{2} + \frac{1}{2} \right) = \frac{A_c^2}{2} + \frac{1}{4} \mu^2 A_c^2$$

$$\text{Power of USB} = \frac{1}{8} \mu^2 A_c^2 = \text{Power of LSB}$$

$$\text{Power of carrier} = \frac{A_c^2}{2}$$

$$\text{Percentage of Power} = \frac{\text{Power of two - side Band}}{\text{total Power}} = \frac{\frac{1}{4}\mu^2 A_c^2}{\frac{A_c^2}{2} + \frac{1}{4}\mu^2 A_c^2} = \frac{\mu^2}{2 + \mu^2}$$

(if $\mu = 1 \text{ perc.} = \frac{1}{3}$)

conclusion

$$\text{total Power of } s(t) = \frac{A_c^2}{2} + \frac{1}{4}\mu^2 A_c^2 = P_c + 2 * P_{SB}$$

$$\text{percentage of power} = \frac{\mu^2}{2 + \mu^2}$$

$$= \frac{\left(\frac{A_c^2}{2} + \frac{1}{4}\mu^2 A_c^2\right)}{R}$$

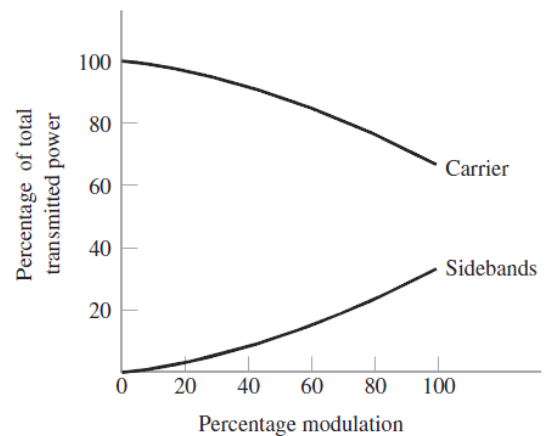
Conditions of AM modulation

$$(1) |K_a m(t)| < 1 \quad (2) f_c \gg \tilde{\omega} \quad \text{Band Width}$$

To prevent distortion or over modulation

The Ratio Between

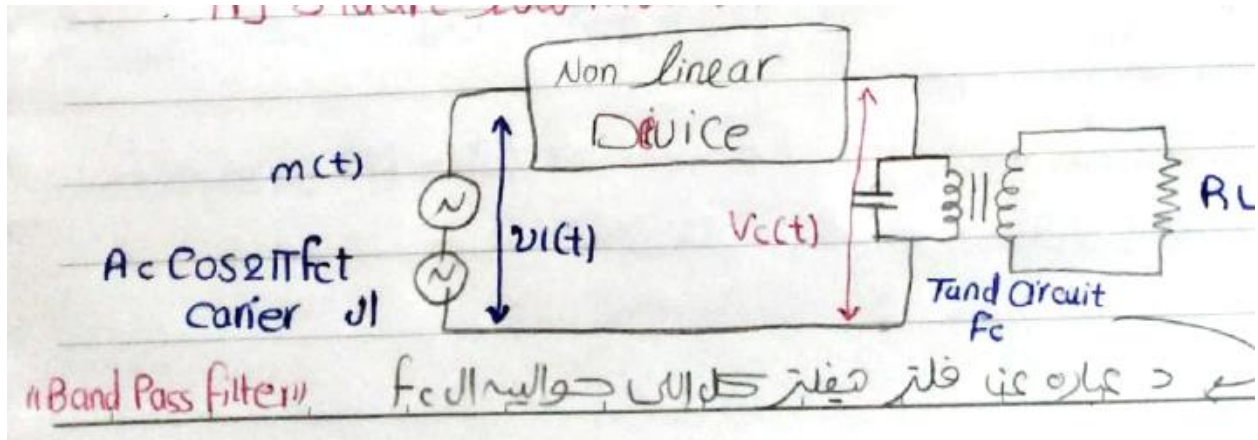
with the increase of μ (percentage modulation) the sidebands power increase while carrier power decrease



GENERATION OF AM WAVES

Modulation

[1] Square Law Modulator



$$\text{output on } R_L \Rightarrow S(t) = A_c[1 + k_a m(t)] \cos(2\pi f_c t)$$

$$\text{Proof output on } R_L \Rightarrow S(t) = A_c[1 + k_a m(t)] \cos(2\pi f_c t)$$

$$\therefore v_1(t) = m(t) + A_c \cos(2\pi f_c t)$$

$$v_2(t) = a_1 v_1(t) + a_2 v_1^2(t) \rightarrow \text{nonlinear device}$$

$$\therefore v_2(t) = a_1[m(t) + A_c \cos(2\pi f_c t)] + a_2[m^2(t) + A_c^2 \cos^2(2\pi f_c t) + 2m(t)A_c \cos(2\pi f_c t)]$$

الحدود التي فيها \cos

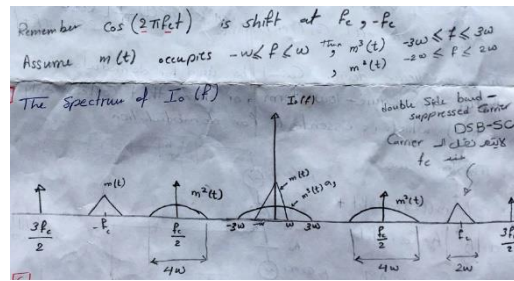
$$\therefore v_2(t) = \cos(2\pi f_c t) [a_1 A_c + 2a_2 A_c m(t)] + \dots$$

$$v_2(t) = a_1 A_c \left[1 + \frac{2a_2}{a_1} m(t) \right] \cos(2\pi f_c t) + \dots$$

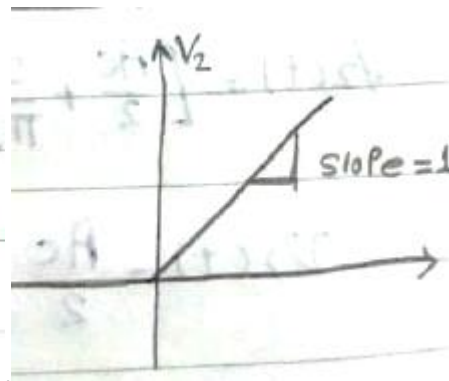
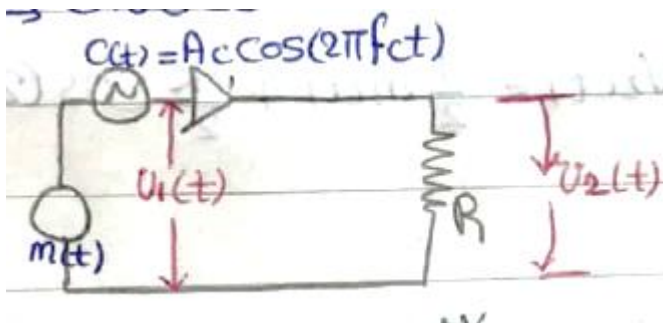
$$a_1 = 1, k_a = \frac{2a_2}{a_1}$$

The filter is to eliminate the unwanted terms

Figure 1 THIS IS FOR $v + v^3$



[2] Switching Modulator



Uses ideal switch (diodes)

forward $\rightarrow c(t) > 0$

reverse $\rightarrow c(t) < 0$

$$v_1(t) = m(t) + A_c \cos(2\pi f_c t)$$

$$\text{at } c(t) > 0 \quad v_2(t) \cong v_1(t)$$

$$\text{at } c(t) < 0 \quad v_2(t) = \text{zero}$$

$$\text{output} \Rightarrow v_2(t) \cong [m(t) + A_c \cos(2\pi f_c t)]g_p(t)$$

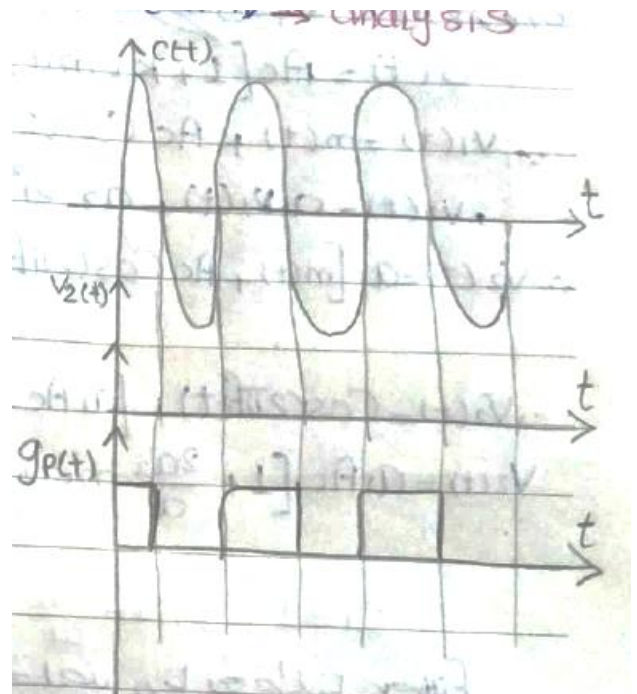
Prove that $g_p(t)$

$$= \frac{1}{2} + \frac{2}{\pi} \sum \frac{(-1)^{n-1}}{2n-1} \cos(2\pi f_c (2n-1)t)$$

$$\text{where } T_c = \frac{1}{f_c}$$

\therefore the function is even $b_n = 0$

$$\therefore a_0 = \frac{1}{T_c} \int_{-\frac{T_c}{4}}^{\frac{T_c}{4}} (1) dt = \frac{1}{T_c} * 2 * \frac{T_c}{4} = \frac{1}{2}$$



$$\begin{aligned}\therefore a_n &= \frac{1}{T_c} \int_{-\frac{T_c}{4}}^{\frac{T_c}{4}} \cos\left(\frac{2\pi nt}{T_c}\right) dt \\ &= \frac{1}{T_c} * \frac{T_c}{2\pi n} \left[\sin\left(\frac{2\pi nt}{T_c}\right) \right] \\ &= \frac{1}{2\pi n} * \sin\left(\frac{2\pi}{2}\right) * 2 = \frac{1}{\pi n} \sin\left(\frac{n\pi}{2}\right)\end{aligned}$$

$$\text{at } n \rightarrow \text{even} \rightarrow a_n = 0$$

$$n \rightarrow \text{odd} \rightarrow a_n = \frac{1}{\pi n} (-1)^{n-1}$$

so

$$g_p(t) = \frac{1}{2} + \frac{2}{\pi} \sum \frac{(-1)^{n-1}}{2n-1} \cos[2\pi f_c(2n-1)t]$$

where $(2n-1) = 1, 3, 5, \dots$

$$\text{Proof output} \Rightarrow v_2(t) \cong [m(t) + A_c \cos(2\pi f_c t)] g_p(t)$$

$$v_2(t) = [m(t) + A_c \cos(2\pi f_c t)] \left[\frac{1}{2} + \frac{2}{\pi} \cos(2\pi f_c t) + -\frac{1}{3} \left(\frac{2}{\pi} \cos(2\pi f_c 3t) \right) + \dots \right]$$

$$v_2(t) = \frac{1}{2} m(t) + \frac{A_c}{2} \cos(2\pi f_c t) + \frac{2}{\pi} m(t) \cos(2\pi f_c t) + \frac{2}{\pi} A_c \cos^2(2\pi f_c t) + \dots$$

$$v_2(t) = \left[\frac{A_c}{2} + \frac{2}{\pi} m(t) \right] \cos(2\pi f_c t)$$

$$v_2(t) = \frac{A_c}{2} \left[1 + \frac{4m(t)}{\pi A_c} \right] \cos(2\pi f_c t)$$

Demodulation of AM wave

[1] Square Law detector

$$v_2(t) = a_1 v_1(t) + a_2 v_1^2(t)$$

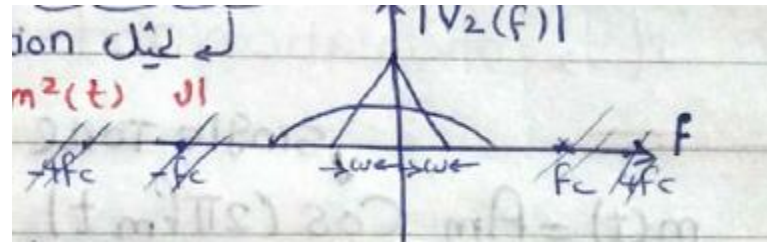
$$v_1(t) = A_c [1 + k_a m(t)] \cos(2\pi f_c t)$$

$$v_2(t) = a_1 A_c [1 + k_a m(t)] \cos(2\pi f_c t) + a_2 * (A_c^2 [1 + k_a m(t)])^2 \cos^2 2\pi f_c t$$

$$v_2(t) = a_1 A_c [1 + k_a m(t)] \cos(2\pi f_c t) + [1 + 2k_a m(t) + m^2(t)k_a^2] * a_2 A_c^2 \left[\frac{1}{2} + \frac{1}{2} \cos 4\pi f_c t \right]$$

اللي مفهوش COS

$$= \frac{1}{2} a_2 A_c^2 (2k_a m(t)) + \overbrace{\frac{1}{2} a_2 A_c^2 k_a^2 m^2(t)}^{\text{Distortion coef of } m^2(t)} + \dots$$



ω = low path filter's width

$$\text{Ratio} = \frac{\text{useful info}}{\text{distortion}} = \frac{2k_a m(t)}{k_a^2 m^2(t)} = \frac{2}{k_a m(t)}$$

Then, for better detection for square law detector

- (1) Weak signal $m(t) \downarrow$
- (2) Small modulation percentage $k_a \downarrow$

Because of it's distortion we have another demodulation circuit

[2] Envelope detector deModulation

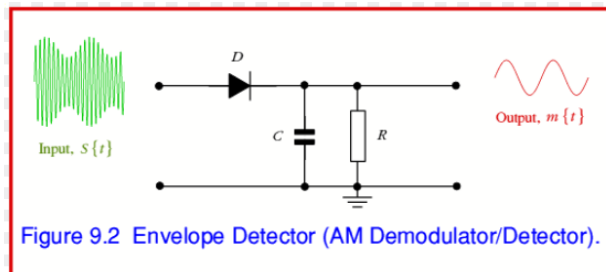


Figure 9.2 Envelope Detector (AM Demodulator/Detector).

$$D \rightarrow \text{on} \rightarrow S.C \rightarrow s(t) = v_{out} \rightarrow s(t) > 0$$

$$D \rightarrow \text{off} \rightarrow O.C. \rightarrow \text{discharging} \rightarrow s(t) < 0$$

$$v(t) = v_m e^{-\frac{t}{\tau}}$$

if $C \uparrow \uparrow$ then will be a line won't have data

if $C \downarrow \downarrow$ will suffer from heavy ripple on data

So it's value has limitations

Charging Condition	Discharging Condition
$\frac{1}{R_L f_c} \leq C < \frac{1}{R_L \omega}$	$C \leq \frac{1}{R_s f_c}$

Defining : Product modulation

$$S(t) = C(t)m(t)$$

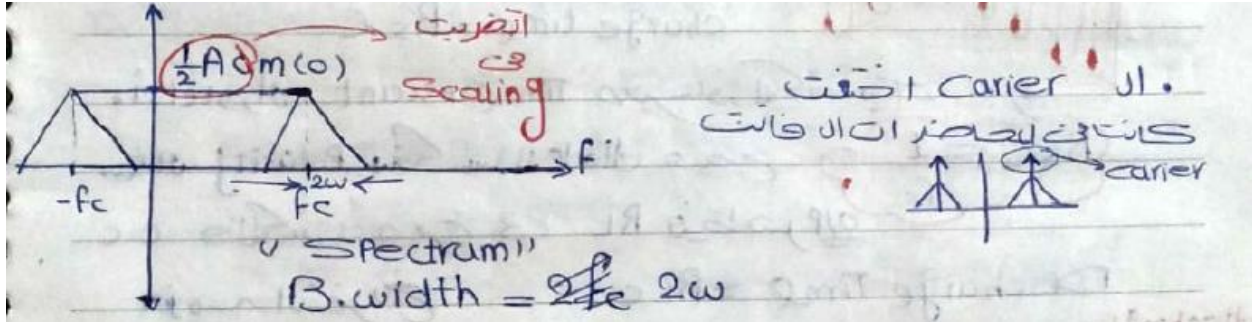
$$S(t) = A_c \cos(2\pi f_c t) m(t)$$

ضرب اشارتين في بعض معناها ان السريعه تكون داخل البطيئه

بشرط عند ال crossing zero يحصل phase reverse

مميزه : المعلومه اللي بدور عليها معدتش ال envelope..

$$S(t) = \frac{1}{2} A_c [e^{j2\pi f_c t} + e^{-j2\pi f_c t}] m(t)$$

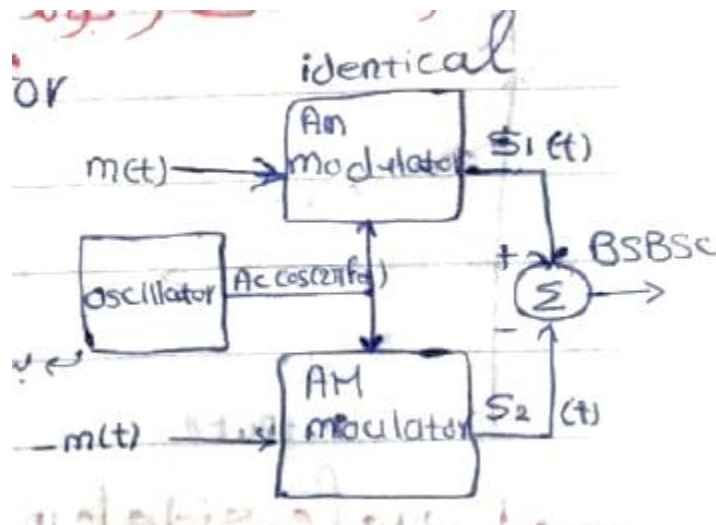


$$\text{BandWidth} = 2W$$

Double Side Band Suppressed Carrier (DSB-SC)

Modulation

[1] Balanced Modulator



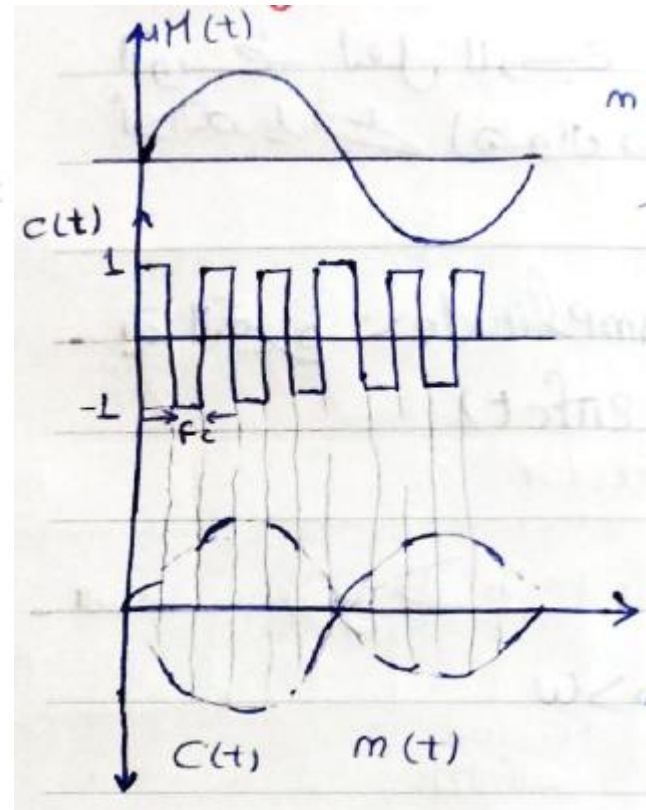
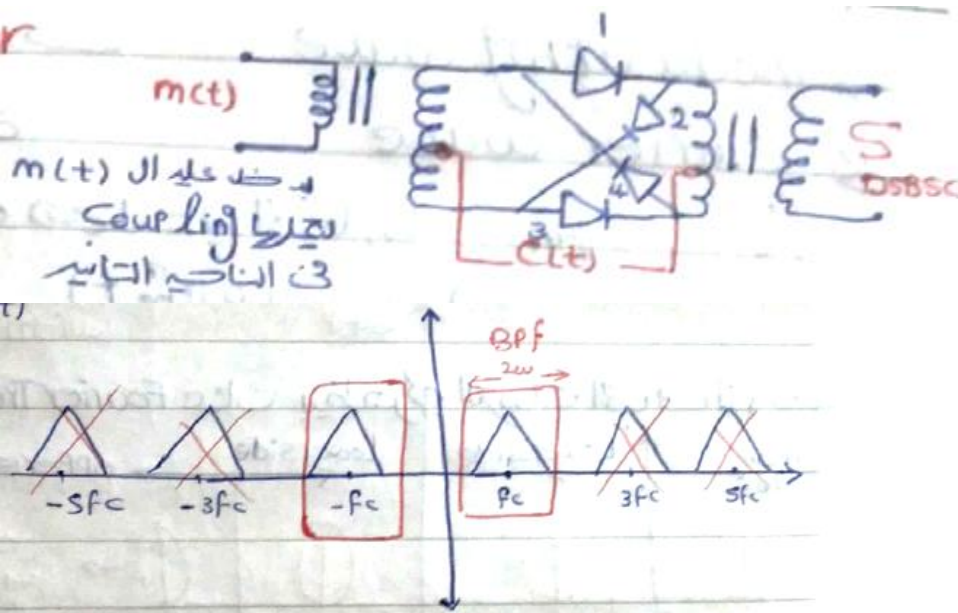
$$S_1(t) = A_c [1 + k_a m(t)] \cos(2\pi f_c t)$$

$$S_2(t) = A_c [1 - k_a m(t)] \cos(2\pi f_c t)$$

$$\begin{aligned} S_{DSBSC}(t) &= S_1(t) - S_2(t) \\ &= A_c \cos(2\pi f_c t) + A_c k_a m(t) \cos(2\pi f_c t) \end{aligned}$$

$$\begin{aligned}
 & -A_c \cos(2\pi f_c t) + A_c k_a m(t) \cos(2\pi f_c t) \\
 & = 2A_c k_a m(t) \cos(2\pi f_c t)
 \end{aligned}$$

[2] Ring Modulator



$$C(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos(2\pi f_c t (2n-1))$$

$$\therefore a_1 = \frac{4}{\pi} \rightarrow f_c$$

$$a_2 = -\frac{4}{3\pi} \rightarrow 3 f_c$$

الاشارة اللي هنتولد هي $m(t) * c(t)$

Prove that $C(t) = \frac{4}{\pi} \sum \frac{(-1)^{n-1}}{2n-1} \cos(2\pi f_c t (2n-1))$

$$\because a_n \rightarrow \text{even} \therefore b_n = 0$$

$$\because a_o = \frac{1}{T_c} \left[\int_{-\frac{T_c}{2}}^{-\frac{T_c}{4}} (-1) dt + \int_{-\frac{T_c}{4}}^{\frac{T_c}{4}} (1) dt + \int_{\frac{T_c}{4}}^{\frac{T_c}{2}} (-1) dt \right] = 0$$

$$\because a_n = \frac{1}{T_c} * \left[\int_{-\frac{T_c}{2}}^{-\frac{T_c}{4}} -\cos\left(\frac{2\pi n t}{T_c}\right) dt + \int_{-\frac{T_c}{4}}^{\frac{T_c}{4}} \cos\left(\frac{2\pi n t}{T_c}\right) dt - \int_{\frac{T_c}{4}}^{\frac{T_c}{2}} \cos\left(\frac{2\pi n t}{T_c}\right) dt \right]$$

after solving

$$\therefore \text{finally, } a_n = \frac{2}{\pi n} \sin\left(\frac{2\pi}{n}\right)$$

therefore,

$$\text{at } n \rightarrow \text{even} \rightarrow a_n = 0$$

$$n \rightarrow \text{odd} \rightarrow a_n = \frac{2}{\pi n} (-1)^{n-1}$$

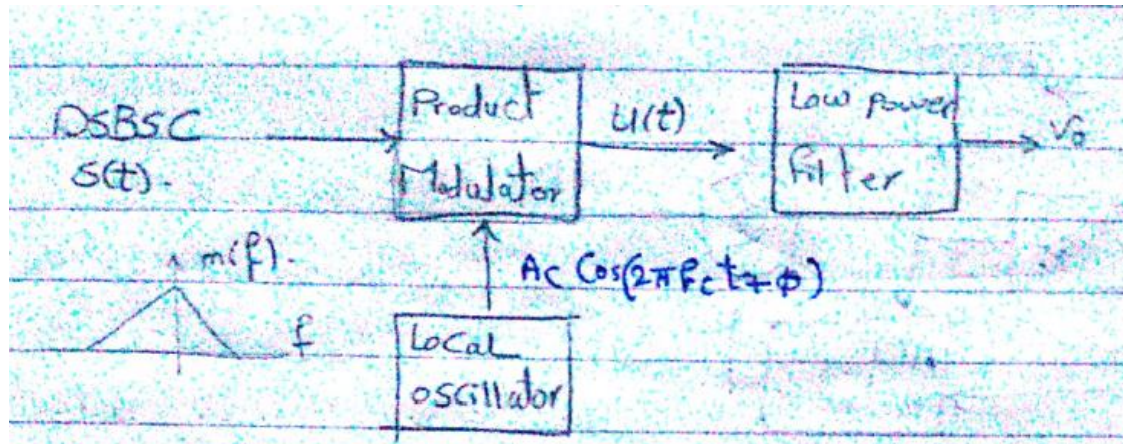
So,,

$$C(t) = \frac{4}{\pi} \sum \frac{(-1)^{n-1}}{2n-1} \cos(2\pi f_c t (2n-1)t)$$

where , $2n-1 = 1, 3, 5, \dots$

Demodulation

[1] Coherent Detection of DSBSC



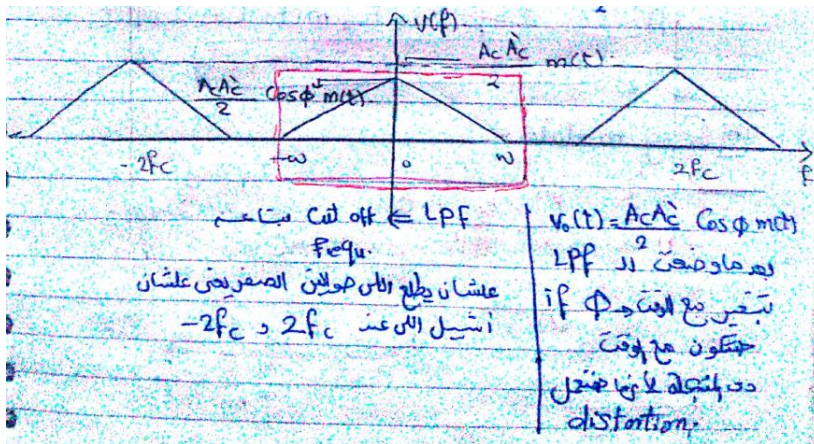
$$s(t) = C(t) * m(t) = A_c \cos(2\pi f_c t) m(t)$$

$$V(t) = S(t) * C(t)$$

$$= A_c \cos(2\pi f_c t) m(t) * A'_c \cos(2\pi f_c t + \phi)$$

$$= \frac{A_c A'_c}{2} m(t) [\cos(4\pi f_c t + \phi) + \cos \phi]$$

$$= \frac{A_c A'_c}{2} m(t) \cos(4\pi f_c t + \phi) + \frac{A_c A'_c}{2} m(t) \cos \phi$$



$$\begin{aligned}
 V(t) &= S(t) \cdot C'(t) \\
 &= A_c \cos(2\pi f_c t) \cdot m(t) \cdot A'_c \cos(2\pi f_c t + \phi) \\
 &= \frac{A_c A'_c}{2} m(t) [\cos(4\pi f_c t + \phi) + \cos \phi] \\
 &\quad \text{After LPF it remove the high Freq Component} \\
 O(t) &= \frac{A_c}{2} m(t) \cos(2\pi \Delta f t)
 \end{aligned}$$

Figure 2 PHASE ERROR

To avoid quadrant null effect

- We use **PLL (Phase Locked Loop)** to produce carrier with a **specific phase**
- Or Sends with the (info signal) one version of the carrier (**Pilot Carrier**)

Disadvantages :

- 1- For Local osc. (if $\phi = 90$) orthogonal then $\cos \phi = 0$ (no information will be transmitted)
[**Quadrant Null Effect**]

- 2- Change in ϕ the signal will cause distortion
- 3- Complex and more costly to make receivers

Advantage

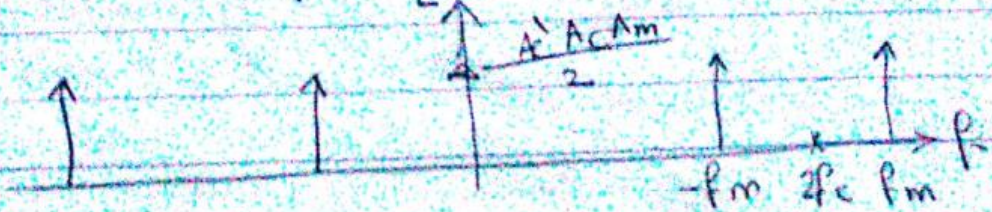
Limited power for the same bandwidth

Ex: $m(t) = A_m \cos 2\pi f_m t$

$$s(t) = A_c A_m \cos(2\pi f_m t) = \frac{A_c A_m}{2} \cos(2\pi f_c + f_m)t + \frac{A_c A_m}{2} \cos(2\pi(f_c - f_m)t)$$

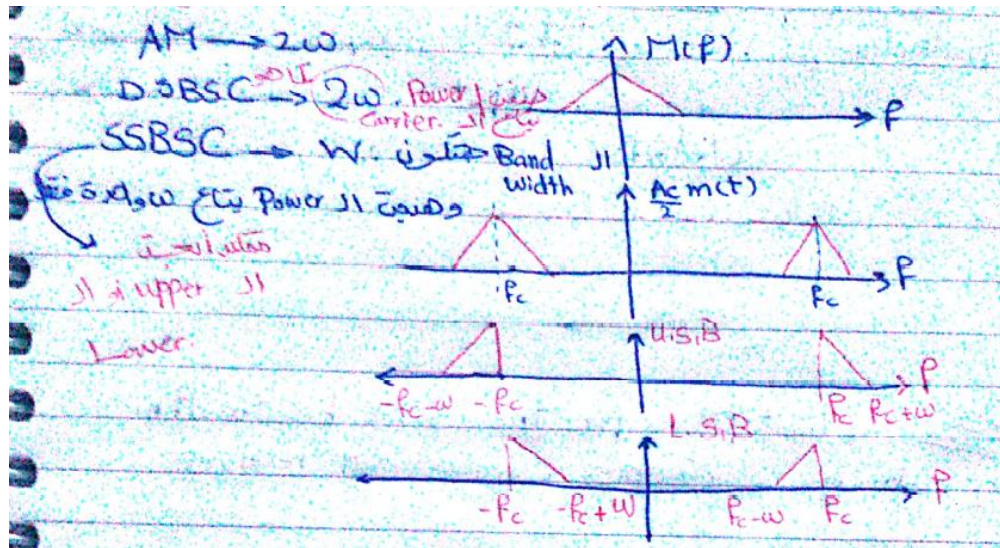
→ Solution

$$v(t) = A_c \cos(2\pi f_c t) \cdot \frac{A_c A_m}{2} \cos(2\pi(f_c + f_m)t) + A_c \cos(2\pi(f_c - f_m)t)$$

$$= \frac{A_c A_c A_m}{4} \left[\cos(2\pi(2f_c + f_m)t) \cos 2\pi f_m t \right] + \frac{A_c A_c A_m}{4} \left[\cos(2\pi(2f_c - f_m)t) + \cos 2\pi f_m t \right]$$


Single Side band Modulation (SSB)

We can get the info signal from only one single side band (SSB) we send one W instead of 2W and the power will be for only one SB



Merits

- 1- Save in Power
- 2- Save in Band Width

DeMerits

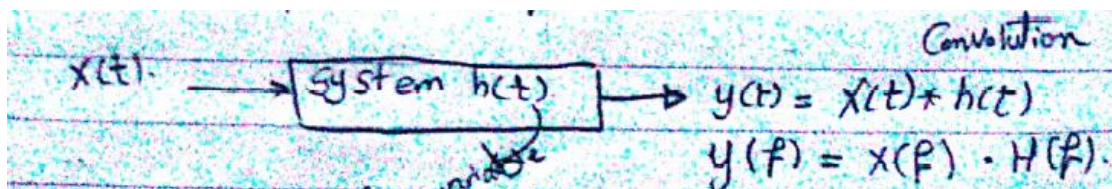
- 1- Complexity
- 2- Costly

Uses

- 1- Vesical Side Band
- 2- In TV and in Cable Phone
- 3- Middle ground between SSB and DSB
- 4- Advanced

How to get a single side band SBS

Note : Hilbert Transfer



A linear time-independent system

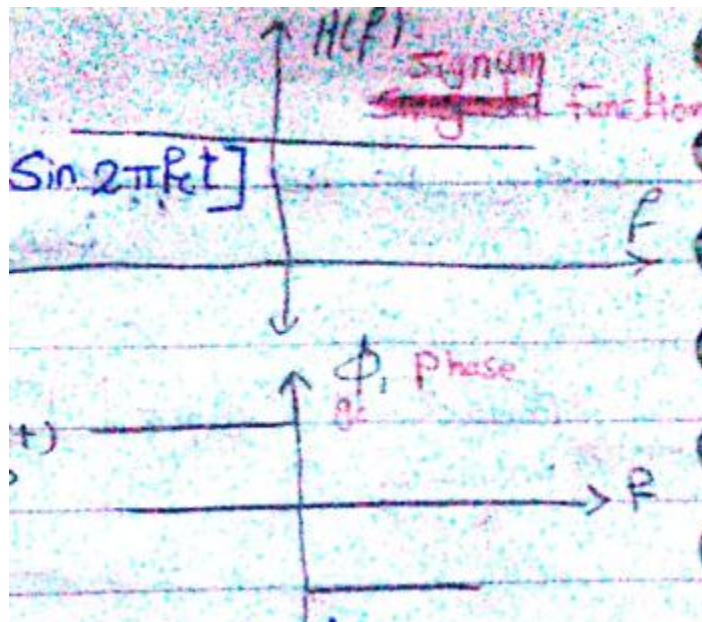
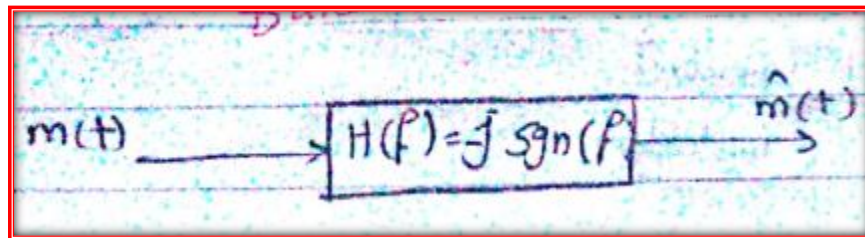
$$s(t) = \frac{A_c}{2} [m(t) \cos(2\pi f_c t) \mp \hat{m}(t) \sin(2\pi f_c t)]$$

– : upper side band

+ : Lower Side Band

$$m(t) = A_m \cos(2\pi f_m t)$$

$$\hat{m}(t) = A_m \sin(2\pi f_m t)$$



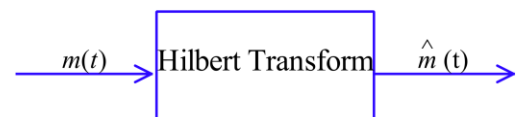
**Prove that $\hat{m}(t) = A_m \sin(2\pi f_m t) \Rightarrow m(t) = A_m \cos(2\pi f_m t)$
Hilbert Transform of $\cos(2\pi f_c t)$ is $\sin(2\pi f_c t)$**

Solution

$$\because \hat{m}(t) = m(t) \otimes h(t)$$

$$h(t) = \frac{1}{\pi t}$$

$$\therefore \hat{M}(f) = M(f) \cdot H(f)$$



$$\text{assume } m(t) = \cos(2\pi f_m t) \Rightarrow M(f) = \frac{1}{2} [\delta(f - f_m) + \delta(f + f_m)]$$

$$\therefore \hat{m}(t) = F^{-1} \{ M(f) \cdot H(f) \}$$

$$\because H(f) = F \left\{ \frac{1}{\pi t} \right\} = j \operatorname{sgn}(-f) = -j \operatorname{sgn}(f)$$

$$\therefore \hat{m}(t) = F^{-1} \left\{ -\frac{j}{2} \operatorname{sgn}(f) [\delta(f - f_c) + \operatorname{sgn}(f) \delta(f + f_c)] \right\}$$

$$\because \operatorname{sgn}(f) \delta(f - f_c) = 1 * \delta(f - f_c)$$

$$\because \operatorname{sgn}(f) \delta(f + f_c) = -1 * \delta(f + f_c)$$

$$\therefore \hat{m}(t) = \frac{1}{2j} F^{-1} \{ \delta(f - f_c) - \delta(f + f_c) \}$$

$$= \frac{1}{2j} * [e^{+i2\pi f_c t} - e^{-i2\pi f_c t}]$$

$$= \sin(2\pi f_m t)$$

Ex: $m(t) = A_m \cos(2\pi f_m t)$.

عام اشرف الإشارة إلى صيغة SSB ولا

$$S(t) = \frac{A_c}{2} [m(t) \cos 2\pi f_c t] + \hat{m}(t) * \sin 2\pi f_c t$$

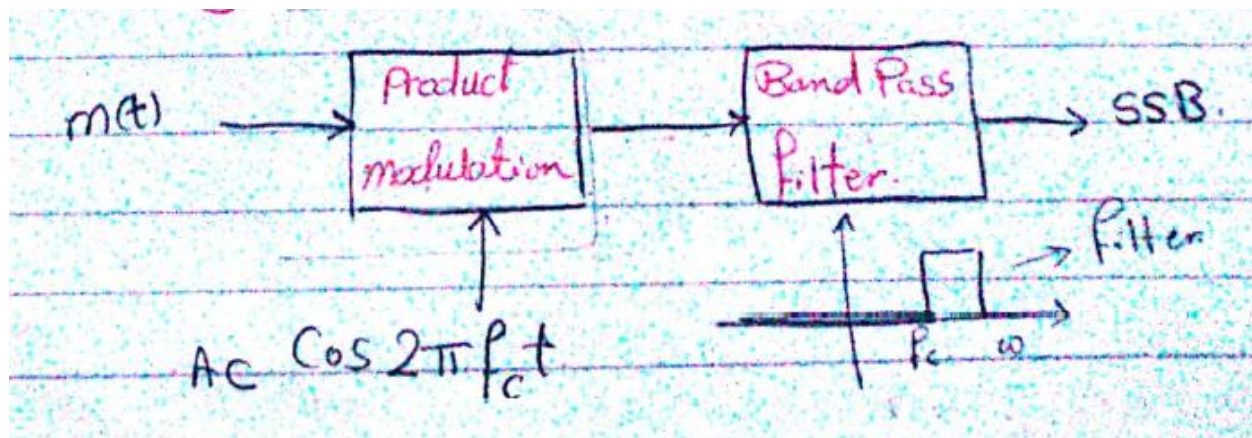
$$\hat{m}(t) = A_m \sin 2\pi f_m t$$

$$S(t) = \frac{A_c}{2} [A_m \cos(2\pi f_m t) \cos 2\pi f_c t + A_m \sin 2\pi f_m t * \sin 2\pi f_c t]$$

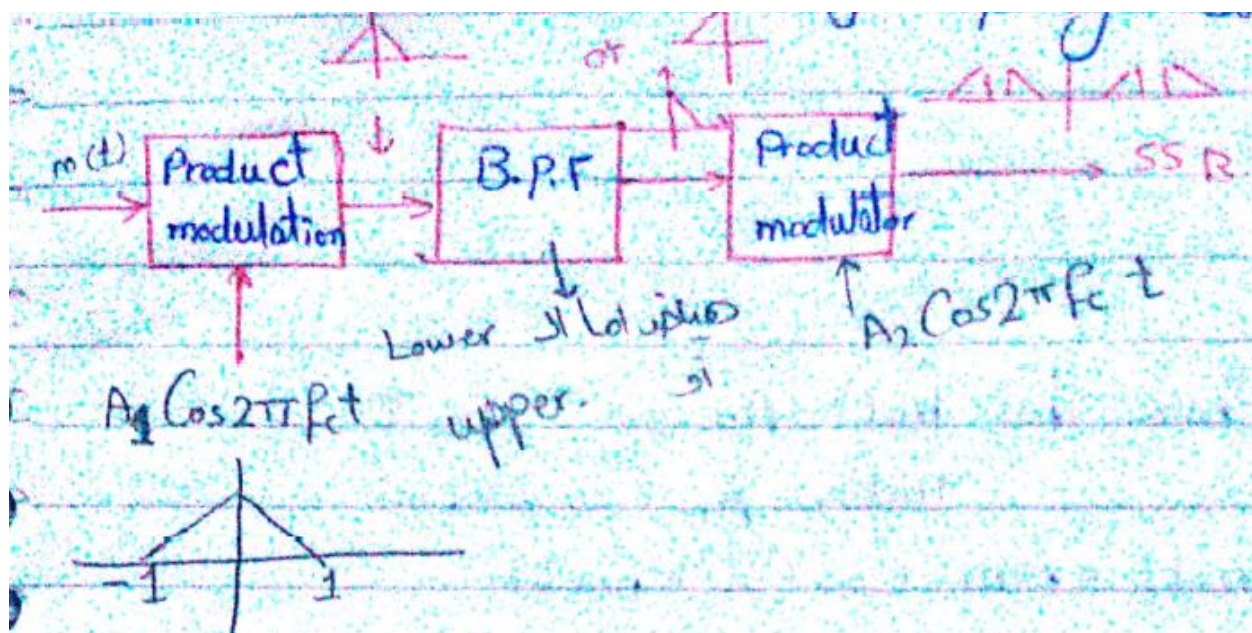
$$S(t) = \frac{A_c A_m}{2} \cos [2\pi (f_c + f_m) t]$$

Modulation

Frequency Discrimination Method

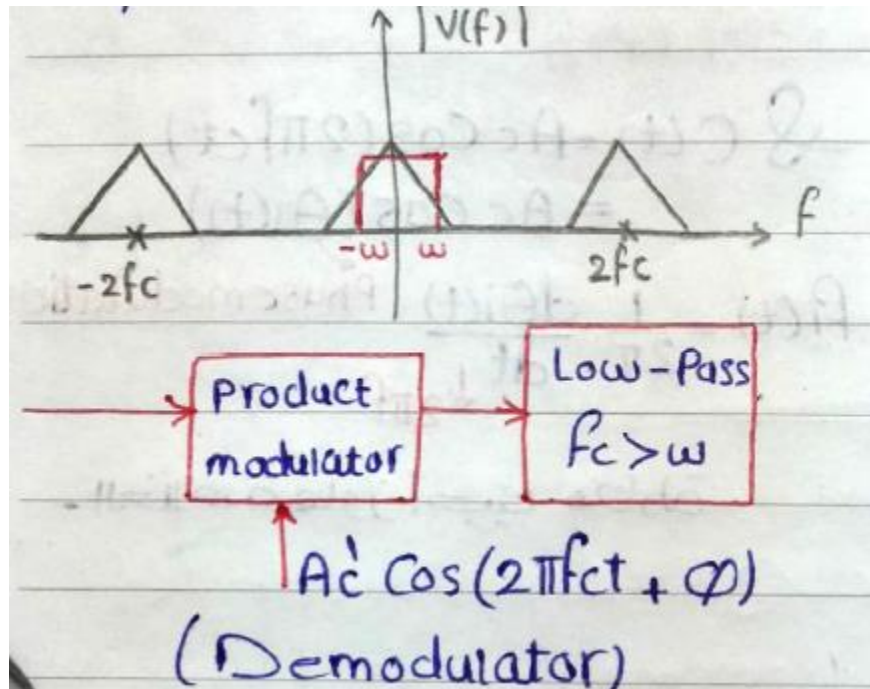


هنصمم Band Pass Filter بيشتغل في ال Upper Side Band فقط ولكن محتاج مكونات كفائتها عاليه



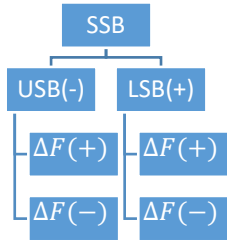
Demodulation

Coherent detection of SSB-SC



$$\begin{aligned}
 S_{SSB}(t) &= \frac{1}{2} A_c [m(t) \cos(2\pi f_c t) \mp m(t) \sin(2\pi f_c t)] \\
 &= \frac{1}{2} A_c A'_c \cos(2\pi f_c t) [m(t) \cos(2\pi f_c t) \mp m(t) \sin(2\pi f_c t)] \\
 &= \frac{1}{4} A_c A'_c m(t) + \frac{1}{4} A_c A'_c m(t) \cos(4\pi f_c t) \mp \frac{1}{4} A_c A'_c m(t) \sin(4\pi f_c t)
 \end{aligned}$$

Errors

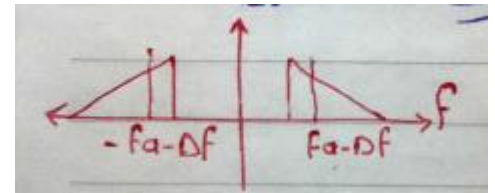
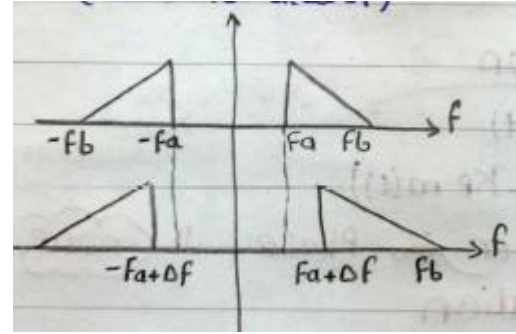


(1) Frequency Error ΔF

نضرب الاشارتين عشان نعرف هنجمع ولا نطرح ال ΔF

In case of human voice – it won't be so significant

Human voice : 400Hz : 4kHz



(2) Phase Error ($\Delta\phi$)

$$v_0(f) = \frac{1}{4} A_c A'_c [M(f) \cos \phi \mp \hat{M}(f) \sin \phi]$$

$$\hat{M}(f) = -j \operatorname{sgn}(f) M(f)$$

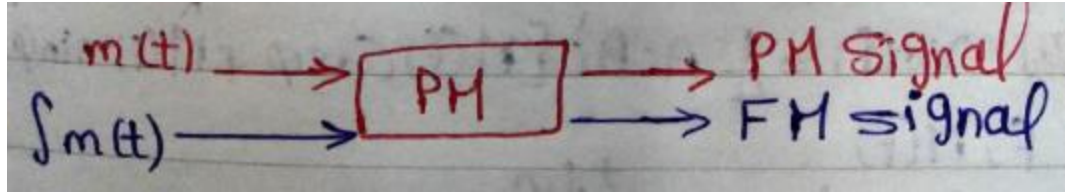
$$V_0(f) = \begin{cases} \frac{1}{4} A_c A'_c M(f) e^{\pm j \phi} \\ \frac{1}{4} A_c A'_c M(f) e^{\mp j \phi} \end{cases}$$

اي انه مش هياثر في ال fr ولكن هيعمل phase ب 90

Angle Modulation	
FM	PM
Frequency Modulation	Phase Modulation
$c(t) = A_c \cos(2\pi f_c t)$	$c(t) = A_c \cos(\theta_i(t))$
Change in frequency (f_i)	Change in phase (θ_i)
$f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt}$	$\theta_i = 2\pi f_c t + \phi$
$f_{i(t)} = f_c + k_f m(t)$ $s(t) = A_c \cos(2\pi(f_c + k_f m(t))t)$ $\theta_i = 2\pi f_c t + \overbrace{2\pi k_f}^{k_p} \int_0^1 m(t) dt$	$\theta_i = 2\pi f_c t + k_p m(t)$ $s(t) = A_c \cos(2\pi f_c t + k_p m(t))$ Note : Phase(θ) Changes Linearly with m(t)

اذن هما وجهين لعمله واحده

وتبص تلاقي



Frequency Modulation

$$S_{FM}(t) = A_c \cos \left(2\pi f_c t + 2\pi k_f \int_0^t m(t) dt \right)$$

Advantages of FM & PM

- 1- **More Security**
- 2- Can carry (one frequency signal) or (multi-tone FM , multiple frequency input as human voice (4Hz to 4kHz)
- 3- Instantons (ΔF)

$$\Delta F, \beta$$

$$\text{instaneous freq. } \widehat{f_i(t)} = \text{Carrier Freq. } \widehat{f_c} + \overbrace{k_f A_m}^{\Delta f} \cos(2\pi f_m t)$$

$$\Delta f = k_f A_m \rightarrow \text{frequency deviation}$$

$$f_{Max} = f_c + \Delta f$$

$$f_{Min} = f_c - \Delta f$$

لو عاوز اجيب ال θ_i هكامل ال $m(t)$ و اضرب في 2π

$$\theta_i(t) = 2\pi f_c + 2\pi k_f A_m \frac{1}{2\pi f_m} \sin(2\pi f_m t) = 2\pi f_c + \overbrace{\frac{k_f A_m}{f_m}}^{\Delta f / f_m = \beta} \sin(2\pi f_m t)$$

$$\beta = \frac{\Delta f}{f_m} = \text{phase deviation}$$

For a sinusoidal (single-tone) fm modulation

$$s(t) = A_c \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$$

According to the value of β FM can be divided to

Narrow Band FM	Wide Band FM
β is small $\downarrow = 0.5 \text{ rad}$ $\text{bandwidth} = 2\omega$ Less quality Δf as $\beta = \frac{\Delta f}{f_m(\text{const.})}$ so we change Δf $\Delta f \downarrow = k_f A_m \downarrow$ So we change $A_m \downarrow \rightarrow \Delta f \downarrow \rightarrow \beta \downarrow$ $\text{BW} = 2(\beta + 1)f_m$ لو تردد الاشارة الاساسيه زاد ممكن يعمل overlap فلازم نعمل clipping	$\beta \uparrow$ $\text{bandwidth} \gg 2\omega$ To produce wide band must produce narrow band لو زاد التردد مش هخاف من ال overlap ومش هحتاج اعمل clipping اقل تغير في ال amplitude هيقابله تغير في التردد عشان كده ال quality افضل

Narrow Band FM

$$s(t)_{FM} = A_c [\cos(2\pi f_c t) * \cos(\beta \sin(2\pi f_m t)) - \sin(2\pi f_c t) \sin(\beta \sin(2\pi f_m t))]$$

$$\text{as } x \downarrow \Rightarrow \sin x \cong x$$

$$\cos(\beta \sin(2\pi f_m t)) \cong 1 \text{ as } \beta \downarrow$$

$$\text{So, } \sin(\beta \sin(2\pi f_m t)) = \beta \sin(2\pi f_m t)$$

$$S(t)_{NB FM} = A_c \cos(2\pi f_c t) * (1) - A_c \beta \sin(2\pi f_c t) \sin(2\pi f_m t)$$

$$\sin A \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$$

$$\begin{aligned} & \sin(2\pi f_c t) \sin(2\pi f_m t) \\ &= \frac{1}{2} [\cos 2\pi(f_c - f_m)t - \cos 2\pi(f_c + f_m)t] \end{aligned}$$

$$S(t)_{NBFM} = A_c \cos(2\pi f_c t) - \frac{A_c \beta}{2} [\cos 2\pi(f_c - f_m)t - \cos 2\pi(f_c + f_m)t]$$

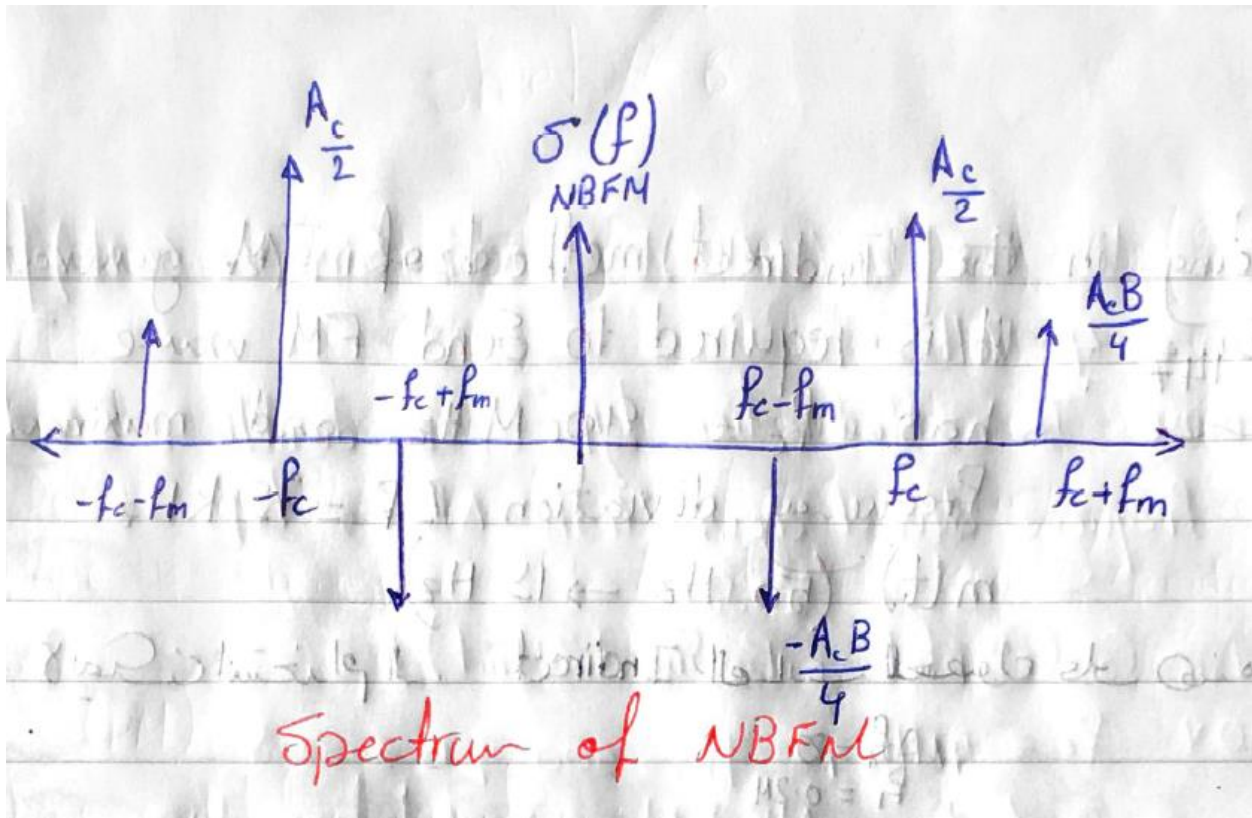
(same Power for any signal, out of phase)

نلاحظ انها تشبه S الخاصه بال AM ولكن فرق الاشارة بين ال cosines

$$S(f)_{NBFM} = \frac{A_c}{2} [\delta(f - f_c) + \delta(f + f_c)] - \frac{A_c \beta}{4} [[\delta(f - (f_c - f_m)) + \delta(f + (f_c - f_m))] - [\delta(f - (f_c + f_m)) + \delta(f + (f_c + f_m))]]$$

$$S_{AM} = A_c \cos(2\pi f_c t) + \frac{\mu A_c}{2} [\cos 2\pi(f_c - f_m)t + \cos 2\pi(f_c + f_m)t]$$

(Power Changes depending on carrier, in phase)



NBPM في حاله ال

(PM)

Phase modulation equation

$$s(t) = A_c \cos(2\pi f_c t + K_p m(t))$$

$$s_{PM}(t) = A_c \cos[2\pi f_c t + K_p A_m \cos(2\pi f_m t)] \quad B = K_p A_m$$

$$= A_c \cos[2\pi f_c t + B \cos(2\pi f_m t)]$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$s_{PM}(t) = A_c \cos 2\pi f_c t \cos(B \cos 2\pi f_m t) - A_c \sin 2\pi f_c t \sin(B \cos 2\pi f_m t)$$

Since B very small < 1

$$\cos(B \cos 2\pi f_m t) = 1$$

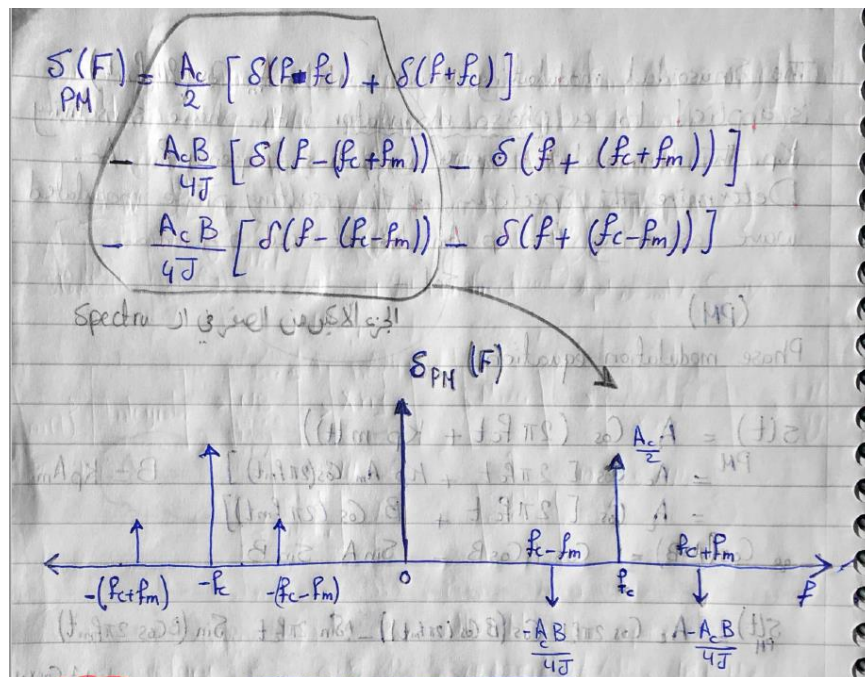
$$\sin(B \cos 2\pi f_m t) = B \cos 2\pi f_m t$$

$$s_{PM}(t) = A_c \cos 2\pi f_c t - A_c \sin 2\pi f_c t B \cos 2\pi f_m t$$

$$\sin X \cos Y = \frac{1}{2} [\sin(X+Y) + \sin(X-Y)]$$

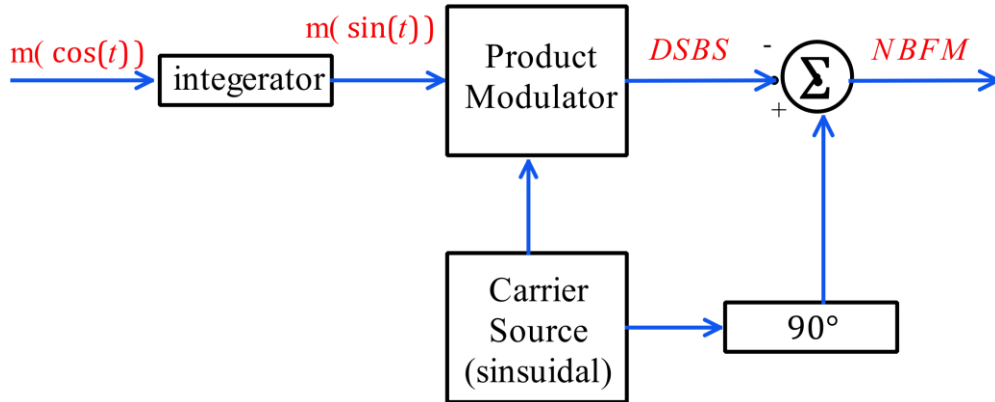
$$s_{PM}(t) = A_c \cos 2\pi f_c t - \frac{A_c B}{2} \sin(2\pi(f_c + f_m)t) - \frac{A_c B}{2} \sin(2\pi(f_c - f_m)t)$$

Fourier Transform Frequency domain



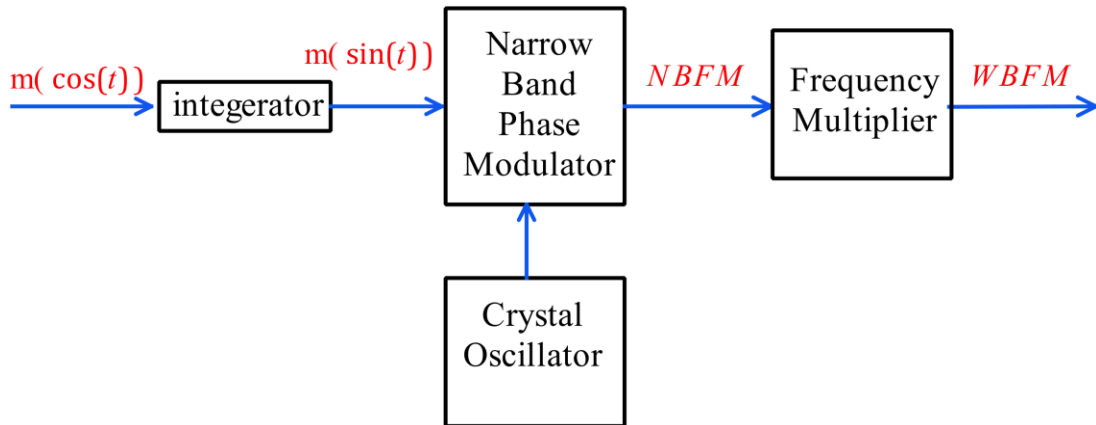
DIAGRAM

Integrator : OpAmp //can also يجمع ويطرح و يفاضل ويكامل



Wide Band FM

How to generate wide-band FM ?



$$S(t) = A_c \cos[2\pi f_1 t + \beta_1 \sin 2\pi f_m t]$$

After frequency multiplier

$$S(t) = A_c \cos[2\pi f_c t + n\beta_1 \sin 2\pi f_m t]$$

ضربت التردد الاصلي بتاع ال Narrow Band عدد n مرات

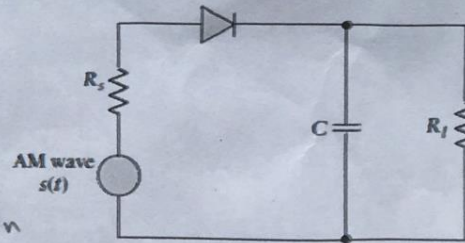
$$\text{where : } f_c = n f_1 , \quad \beta = n \beta_1$$

$$FM \text{ radio} \Rightarrow 88 : 108 \text{ MHz}$$

Examples P,149 ???

- 3) For the envelope detector circuit shown below:
if: $R_S = 75$, $R_L = 10\text{ K}\Omega$, $W = 1\text{ KHz}$, $f_c = 20\text{ KHz}$

Determine a suitable value for C for proper operation
(validate your result).



charging Condition

$$C \leq \frac{1}{R_S f_c} \rightarrow C \leq 6.67 \times 10^{-7} \text{ F}$$

667 nF

discharging Condition

$$\frac{1}{R_L f_c} \leq C \leq \frac{1}{R_L W} \rightarrow 5 \times 10^{-9} \leq C \leq 10^{-7} \text{ F}$$

$5 \text{ nF} \quad 100 \text{ nF}$

$\therefore C = 10^{-8} \text{ F}$

$C = 47.5 \text{ nF}$

$C = 10 \text{ nF}$

WHAT IS THIS

$B = 0.8 < 1$ NBFM modulation type

مش المفروض اخرها تكون 10.5

$$\int_{-\infty}^t g(t) dt \Leftrightarrow \frac{1}{j2\pi f} G(f) + \frac{G(0)}{x} S(f) ??$$

$$g_3(t) = \frac{1}{2} e^{(-\pi^2/4)}$$

$$g(t) = \frac{1}{2} e^{-\pi \left(\frac{t}{2}\right)^2}$$

$$\rightarrow 2 * \frac{1}{2} e^{-\pi (f*2)^2} = e^{-4\pi f^2}$$

$$\rightarrow e^{-4\pi f^2}$$